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Book Reviews

BOOK REVIEWS

Our section starts with the featured review of Glenn Ledder's book *Mathematical Modeling for Epidemiology and Ecology*. This review is a joint work of 10 authors from Anita Layton's group. This shows that one can efficiently combine a reading course with an introduction to scientific work and the writing of a review. All reviewers are enthusiastic about the book and recommend it to all researchers interested in the topic.

We come from biological aspects to Eric Lauga's book *Fluid Mechanics: A Very Short Introduction*. Reviewer Anita Layton recommends the book especially for the younger generation of researchers. This is followed by Guenther Leobacher's review of *An Introduction to the Numerical Simulation of Stochastic Differential Equations*, by Desmond Higham and Peter Kloeden. The reviewer praises in particular that the authors have made the "not so easy subject accessible." After that Guenter Leugering reviews the book *PDE Control of String-Actuated Motion*, by Ji Wang and Miroslav Krstic. This book is recommended as having only a small gap between mathematical rigor and engineering objectives. In contrast to that, Sean M. Eli and Krešimir Josić note that in the book *Visual Differential Geometry and Forms*, by Tristan Needham, "the intuition gained" is "at the expense of the rigor." The final review is written by Mehrshad Sadria. He praises *Probabilistic Machine Learning: An Introduction*, written by Kevin Murphy, as a "valuable asset for both beginners and experienced readers."

On a final note, after six years of service, I am pleased to pass on the responsibility of the Book Reviews section to Anita Layton, who has been a valued member of our editorial board for some time now. I am delighted to announce that starting in 2024, Anita will be taking the helm of this highly successful segment of *SIAM Review*. I wish her as much joy and fulfillment in this role as I have experienced.

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Book Reviews

Edited by Volker H. Schulz

Featured Review: Mathematical Modeling for Epidemiology and Ecology. Second Edition. *By Glenn Ledder.* Springer Nature, Cham, Switzerland, 2023. $59.99. xx+ 364 pp., hardcover. ISBN 978-3-031-09453-8.

Mathematical Modeling for Epidemiology and Ecology, authored by Glenn Ledder, an Emeritus Professor of Mathematics at the University of Nebraska–Lincoln, presents readers with the necessary mathematical tools to comprehend and utilize mathematical models; it also delves into advanced mathematical biology literature. More specifically, the book explores the application of mathematics in biological contexts, specifically ecology and epidemiology, emphasizing key mathematical concepts and their biological implications through comprehensive explanations. The author assumes no prior mathematical background beyond elementary differential calculus.

The structure of the book is as follows: It begins with an introductory chapter that covers the fundamental principles of mathematical modeling. Subsequent chapters delve into empirical modeling and mechanistic modeling, providing a thorough treatment of essential ideas and techniques often overlooked in mathematics texts, such as the Akaike Information Criterion (AIC). The latter half of the book concentrates on the analysis of dynamical systems, focusing on simplification techniques for analysis, including the Routh–Hurwitz conditions and asymptotic analysis. Instructors have the flexibility to structure courses around either the first or second half of the book, or choose thematically from both sections, such as offering a course on mathematical epidemiology. The biological content within the book is self-contained and encompasses various topics in epidemiology and ecology. Some of this material is presented through case studies that explore detailed examples, while others draw on the author's recent research on vaccination modeling and scenarios from the COVID-19 pandemic.

We will provide comments on individual chapters below. But what's common among chapters is that each contains problem sets that include interconnected problems that present a single biological setting in multistep scenarios, categorized into relevant sections. This approach allows readers to gradually develop comprehensive investigations into topics such as HIV immunology and the sustainable harvesting of natural resources. Some problems incorporate computer programs developed by the author using MATLAB or Octave, complementing more traditional mathematical exercises and equipping students with a comprehensive set of tools for model analysis. Each chapter includes additional case studies in the form of projects, accompanied by detailed instructions. Additionally, appendices provide mathematical details on optimization, numerical solution of differential equations, scaling, linearization, and advanced utilization of elementary algebra to simplify problems.

Chapter 2 provides a comprehensive exploration of *empirical modeling*, a methodology that involves the application of mathematical models to determine the optimal

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parameter values that yield the best fit for the collected data. The chapter gradually progresses from elementary linear mathematical models to more complex nonlinear models. The chapter begins by introducing the method of least squares, and explains how to use it to fit linear models (e.g., $y = mx + b$) and how to linearize selected nonlinear models. While we understand that the author assumes only elementary mathematical background, if a reader is unfamiliar with a straight line, how much of this book can they truly comprehend? The lack of mathematical maturity is often a bigger obstacle than the lack of training itself. Putting that concern aside, we note that the chapter builds upon those foundations and proceeds to develop semilinear models that offer a generalized representation of the exponential and power function models, accommodating any model expressed as $y = A(x; p)$, with p representing the parameter of the nonlinear function $f(x)$. This extension allows for greater flexibility in modeling a wide range of phenomena.

Moreover, the chapter highlights the importance of selecting a model that matches the scenario. That important concept in modeling is explained using simple linear models. We also appreciate the author's discussion of the principle of Occam's razor, which advocates for choosing the simplest model whenever feasible. The author provides comprehensive examples to demonstrate how a simple model can predict better than a complex model. To quantitatively determine the most suitable model, the AIC is employed, providing a rigorous framework for model selection. Here, the author could have included an explanation behind the replacement of the statistical parameter K in the AIC equation with $(k + 1)$, where k denotes the number of parameters in the model.

Lastly, the chapter concludes by providing a case study of different methods used to fit the parameters of the nonlinear Michaelis–Menten model. The author compares different linearization schemes used to fit this model through their AIC scores and explains why some of these methods have inferior performance. This gives the reader a hands-on experience of fitting a model that is extensively used in biological modeling.

In conclusion, Chapter 2 is well designed for beginners in empirical modeling, showcasing a meticulous approach to methodological development through mathematical proofs, MATLAB examples, and graphical illustrations. By incorporating exercises and examples, the author facilitates active learning and comprehension. The use of graphical elements proves to be a valuable tool for conveying complex ideas concisely and intuitively. To further enhance the academic review, the inclusion of a critical analysis of the methods would provide readers with a more nuanced perspective, enabling them to assess the strengths and weaknesses of the presented techniques.

Having developed some elementary modeling tools, in Chapter 3 the author introduces how to formulate mechanistic models. This class of models is based on the fundamental laws of natural sciences, including physical and biochemical principles. By capturing fundamental principles of a system, one may obtain explanatory value from the components of the model. As in other chapters, the author conveys this information in manageable bites, using a simple, clear, and concise writing style. The overall goal of the chapter is to explain how to build mechanistic models with different levels of complexity, using simple tools covered in Chapter 2 and new ones introduced here, and to illustrate these concepts using examples in epidemiology.

One topic that is covered is compartmental analysis, which is a useful approach for providing large-scale structure for some models. Here, the Susceptible-Exposed-Infected-Recovered (SEIR) epidemic model is used as an example. Specifically, the authors introduce the basic reproduction number (a concept that a good fraction of the general public has become familiar with during the pandemic), analytical and numerical methods, early-phase exponential growth, and the end state of such models. The popularity of SIR-type models has skyrocketed during the pandemic, rendering them particularly helpful as examples here. Equivalent forms of mechanistic models are also described and discussed. The author notes that some mechanistic models may be written in different forms based on one's preference and analysis type. The author also shows how the reader may identify equivalent models as well as convert their own mechanistic models into other forms such as a dimensionless form.

We find this chapter is a good starting point for those seeking to understand mechanistic modeling with concrete examples, especially in the context of epidemiology and ecology.

Chapter 4 discusses various types of models describing single population dynamics and the analysis that can be done. The author presents the discrete case, the continuous case, and an example of how to approach the analysis. In the discrete case, the author covers a general seasonal population model, the exponential and logistic situations associated with the model, and the cobweb plot for analysis of stability and fixed points. The author then introduces the continuous equivalents, showing the methods to find equilibrium points, visualize behavior with phase plots, and linearize a model to find local stability. Finally, they give the example of using the above techniques to analyze whale population dynamics, modeled by a growth-consumer. Doing so enables them to create a narrative.

Overall, we find the choice of topics in this chapter suitable, and the author's writing style particularly appealing to first-time learners in ecology modeling. Because the techniques introduced in this chapter are rather general, they can be transferred to many other use cases of population dynamics analysis, such as animals, diseases, and lake algae.

Chapter 5 moves naturally from single to multiple population dynamics; it delves into the dynamics of systems consisting of multiple related quantities changing over discrete time intervals. It expands upon the concepts discussed in Chapter 4 by introducing a more complex model of several subpopulations. The chapter focuses on linear systems, which are used to represent structured populations categorized by age, size, or developmental stage.

The chapter takes a very hands-on approach to learning. It is full of helpful features aimed at improving the reading experience and learning outcomes. It begins with a list of learning objectives; key definitions are highlighted as they emerge; and breakpoints are introduced to prompt the reader to check their understanding. Importantly, the first technical exposition is a practical example of the evolution dynamics of a juvenile and adult population. The author stresses the use of schematic diagrams to represent word problems. The approach of preceding theory by applications can be helpful for understanding mathematical concepts. It can allow students to develop intuition for the methods, which often aids later understanding of the theory. The language is accessible to a wide-range audience, particularly undergraduate students in mathematics, who are the intended target group. The author wisely advises readers on the importance of parameter estimates for useful predictions, particularly in the context of population dynamics.

Overall, Chapter 5 is well illustrated with three cases ranging from ecology to biology; exercises at the end of each chapter build directly upon the applications presented. This chapter reads well as a self-study text for those interested in learning about modeling in ecology and epidemiology. As a result, it is most relevant to applied undergraduate students, epidemiologists, and theoretical biologists, though more seasoned mathematicians may also find some of the content interesting. Indeed, we have found it to be one of the best introductions to discrete linear systems for beginners.

The book ends with Chapter 6, which focuses on nonlinear dynamical systems with multiple variables, expanding upon the foundation laid in the previous chapter of a single variable world.

The initial portion of the chapter introduces readers to phase plane analysis, an essential tool for studying the stability of equilibria in nonlinear systems. The author skillfully breaks down this concept, making it accessible and comprehensible to readers by providing examples, theorems, and problem sets. The use of practical examples and clear explanations enhances the learning experience and ensures that the material is not overly daunting. The subsequent sections of the chapter focus on two methods of analyzing nonlinear systems: the standard eigenvalue-based approach and the more advanced Routh–Hurwitz conditions. The author adeptly presents these methods, offering a comprehensive understanding of their respective strengths and weaknesses. The inclusion of a case study example enriches the chapter further, providing readers with a practical application that reinforces the theoretical concepts discussed. In addition to covering continuous systems, the chapter also delves briefly into discrete systems and their limitations. The author provides valuable insights into the circumstances in which discrete systems are preferable, particularly within a biological setting.

We will end with comments that apply equally well to Chapter 6 and the book in general: We find this chapter a well-structured and informative segment of the book. As we noted before, the author's writing style is clear and engaging, ensuring that readers can follow along and grasp the intricacies of the subject matter. The combination of theoretical explanations, case study examples, and discussions of practical applications creates a comprehensive learning experience. One minor suggestion for improvement would be to include a code section below each example for readers to practice applying the concepts learned. This might allow readers to solidify their understanding and improve their problem implementation skills.

Our conclusion is short: This is a well-written book, highly suitable for applied math undergraduate students.

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Fluid Mechanics: A Very Short Introduction. *By Eric Lauga.* Oxford University Press, Oxford, 2022. $11.95. 144 pp., softcover. ISBN 978-0-198-83100-6.

I love fluid, and that is why I chose Eric Lauga's *Fluid Mechanics: A Very Short Introduction* to review. And the book most certainly did not disappoint! Upon opening the book, I was immediately captivated by the very first sentence in the preface:

> Fluid mechanics is many things to many people.

Indeed! It is fair to say that most readers of this book are already interested in fluid me-